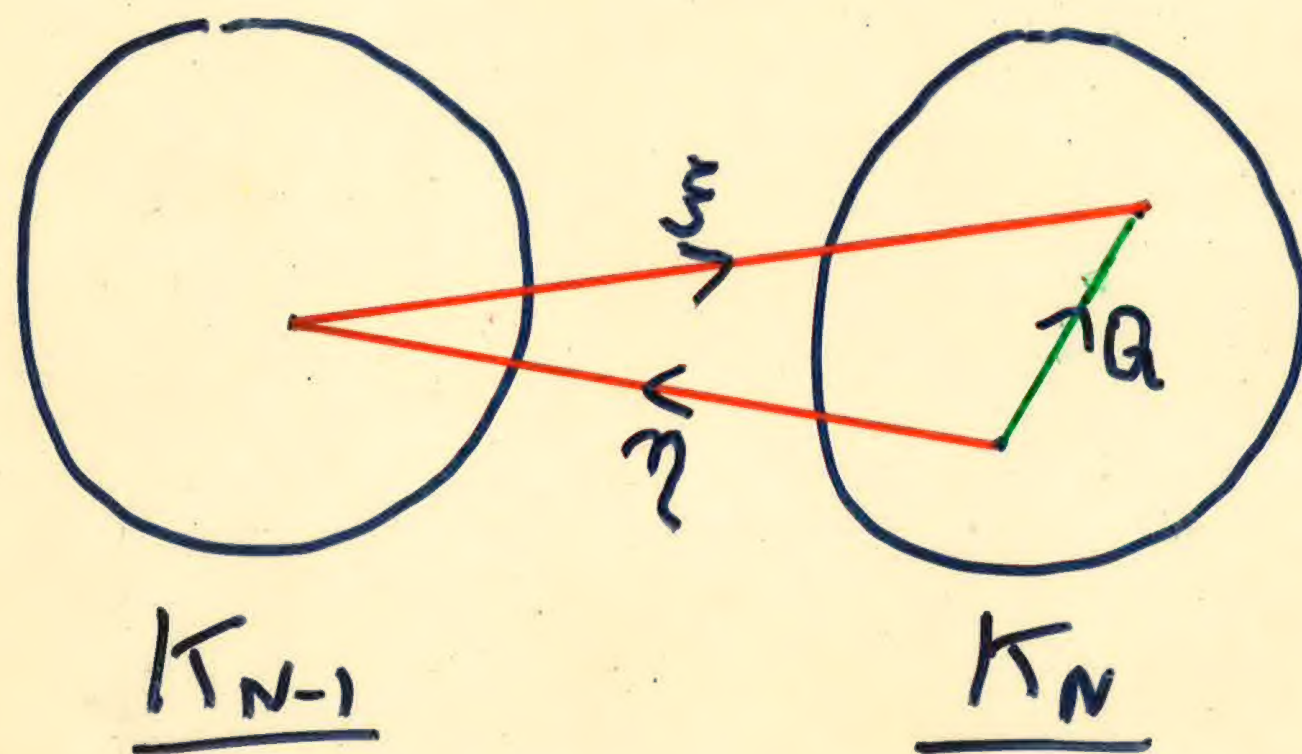


(5a)

Schematically we factorize
 $Q = \xi \cdot \eta$



But $Q' = \xi + \eta$ would create and annihilate particles

FIELD QUANTIZATION

Real Klein-Gordon field

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \mu^2 \right) \psi = 0$$

Energy spectrum

$$E = \sum_{\vec{A}} n_{\vec{A}} (\hbar \omega_{\vec{A}}) + \text{const.}$$

$$\text{where const.} = \frac{1}{2} \sum_{\vec{A}} (\hbar \omega_{\vec{A}})$$

$$\text{and } \omega_{\vec{A}} = c \sqrt{\mu^2 + \vec{k}_{\vec{A}}^2}$$

$n_{\vec{A}}$ are integral eigenvalues of the operator $N_{\vec{A}} = a_{\vec{A}}^\dagger a_{\vec{A}}$

$$\text{where } a_{\vec{A}} |n_{\vec{A}}\rangle = \sqrt{n_{\vec{A}}} |n_{\vec{A}} - 1\rangle$$

$$\text{and } a_{\vec{A}}^\dagger |n_{\vec{A}}\rangle = \sqrt{n_{\vec{A}} + 1} |n_{\vec{A}} + 1\rangle$$

(3c)

So Basic Result of QFT
is :

The Number of particles (quanta)
present with momentum
($\hbar \underline{k}$) and energy ($\hbar \omega_{\underline{k}}$)
in the Particle Representation
is just the excitation Number
 $n_{\underline{k}}$ of the \underline{k} -mode

THE VACUUM

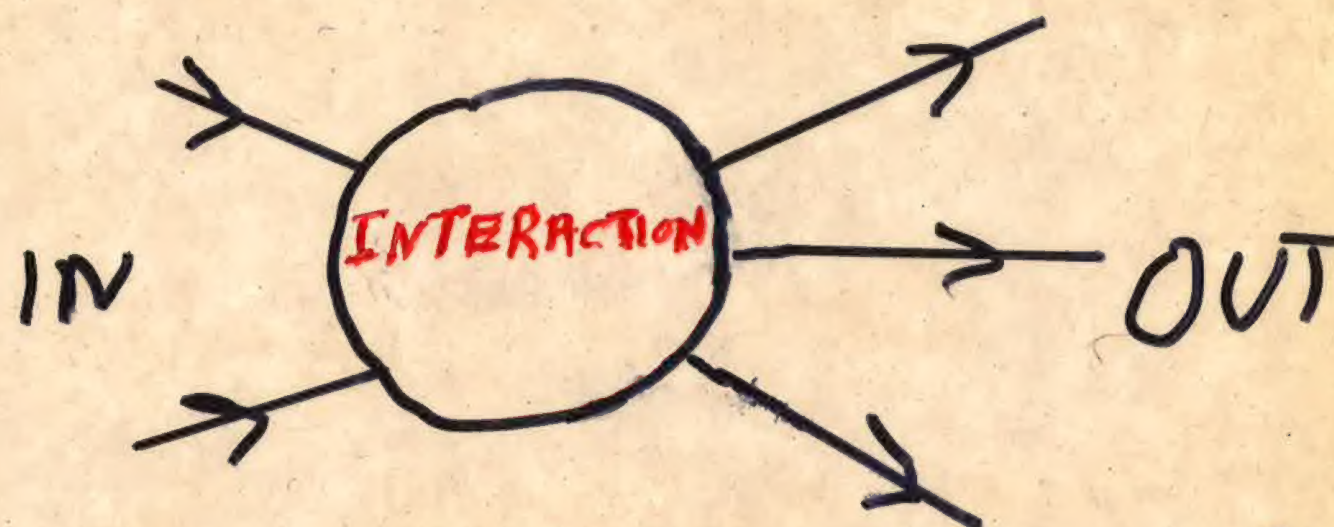
This is the state for which all the n_k are zero.

It is the lowest energy state of the field.

But the energy is not zero, since the field amplitude and other local quantities exhibit Vacuum fluctuations.

The non-vanishing energy of the vacuum is called the zero-point energy of the field.

SCATTERING THEORY



The transition amplitude from the initial IN state to a final OUT state defines the S-Matrix

$$\langle \text{OUT} | S | \text{IN} \rangle$$

$$\text{If } |\psi(t)\rangle = \sum_n c_n(t) |\phi_n\rangle$$

where $|\phi_n\rangle$ are eigenstates
of H_0 , and

$$|\psi(-\infty)\rangle = |i\rangle$$

Then the transition amplitude
to the OUT state $|\phi_n\rangle$
is given by $c_n(\infty)$.

Feynman Diagrams

$$K = K_0 + K_0 V K$$

$$\text{or } (1 - K_0 V) K = K_0$$

$$\text{So } K = (1 - K_0 V)^{-1} K_0$$

$$= \sum_{n=0}^{\infty} (K_0 V)^n \cdot K_0$$

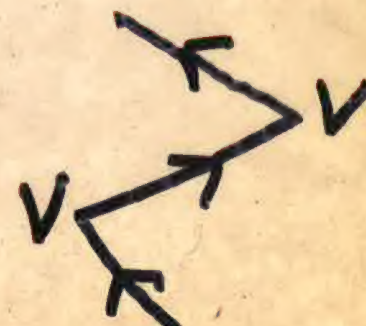
$$= \underline{K_0} + \underline{K_0 V K_0} + \underline{K_0 V K_0 V K_0} + \dots$$



No scattering



Single scattering



Double scattering

$$\langle 2 | K_0 | 1 \rangle \stackrel{\text{def}}{=} K_0(2,1)$$

$$= \sum_n \phi_n^*(x_1) \phi_n(x_2) e^{-i/\hbar E_n (t_2 - t_1)} \cdot \theta(t_2 - t_1)$$

$$\text{where } \theta(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

$K_0(2,1)$ now satisfies the inhomogeneous equation

$$\left(i \frac{\partial}{\partial t_2} - H(x_2)\right) K_0(2,1) = \delta(t_2 - t_1) \cdot \delta(x_2 - x_1)$$

so it is a genuine Green's function.

SECOND QUANTIZATION

Start with N -particle wave

Eq. for an assembly of Bosons.

State is specified by giving n_i
 n_i of particles in i^{th} 1-particle
 state $|u_i\rangle$ (with energy E_i)

Then
$$E = \sum_i n_i E_i$$

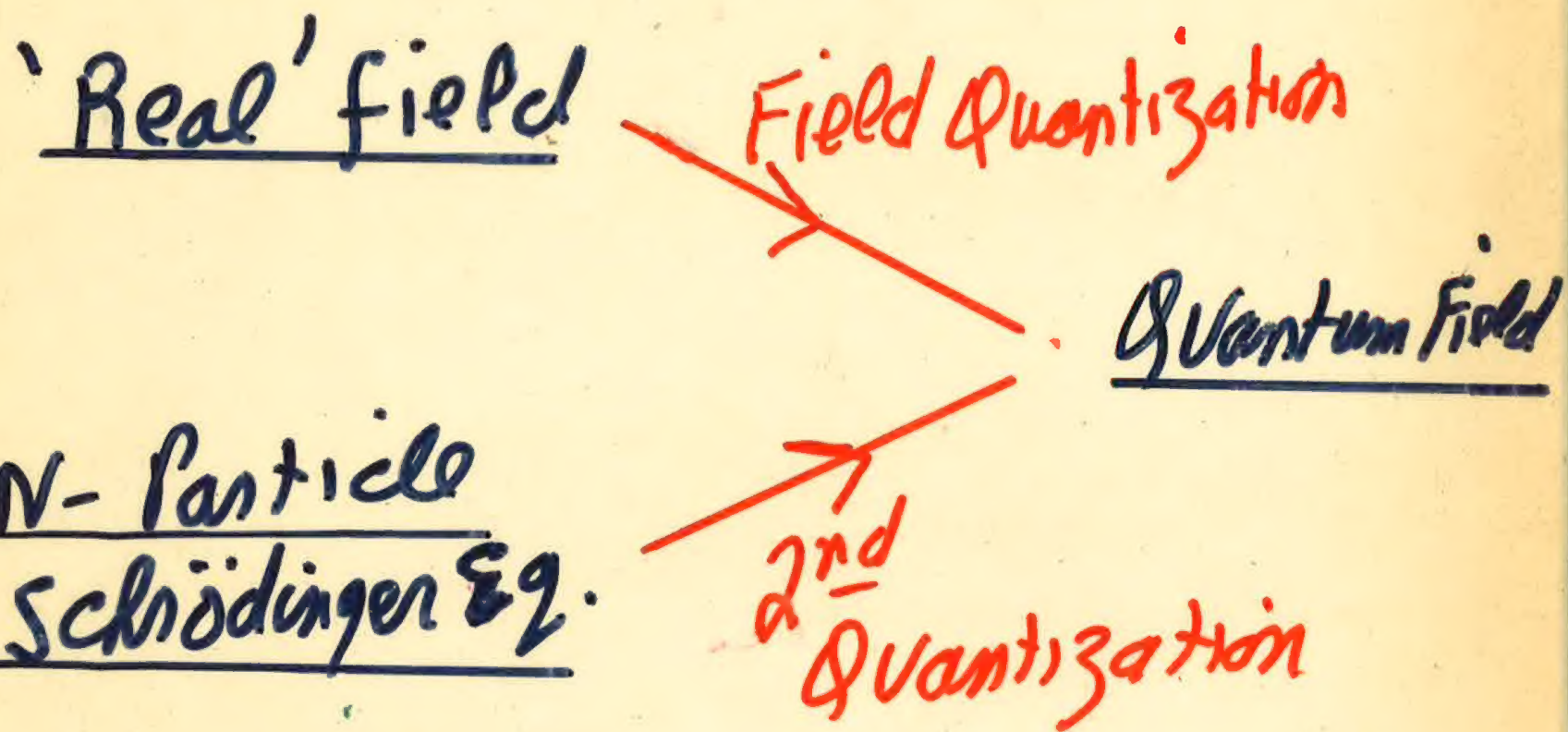
Compare with assembly of harmonic
 oscillators

*
$$E = \sum_i (n_i + \frac{1}{2}) E_i, \text{ if } \omega_i = E_i/\hbar$$

But * is what we would get by subjecting
 the 1-particle W. Eq. to a second
quantization
 (such as K.G. Eq.)

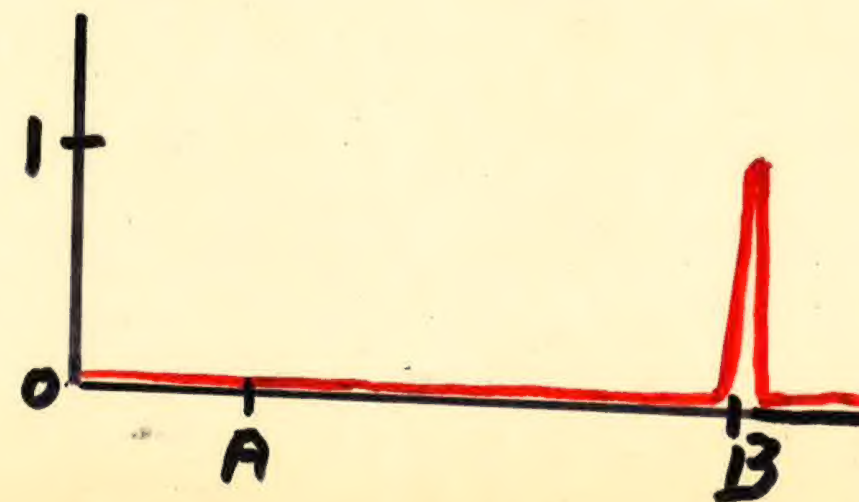
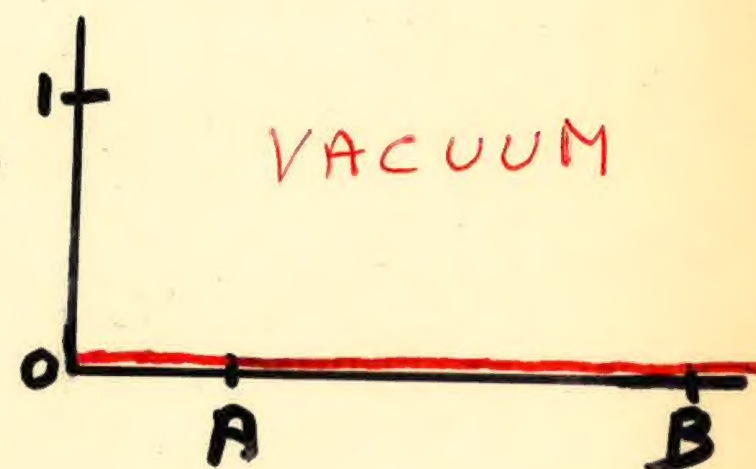
⑦

THE TWO ROUTES TO QFT

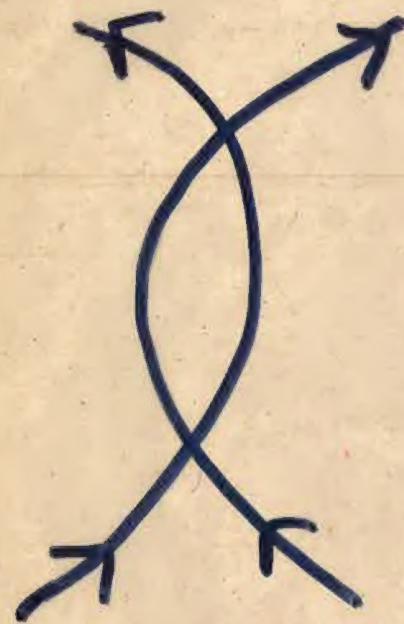


Query : Is Quantum Field the same animal in the two cases?

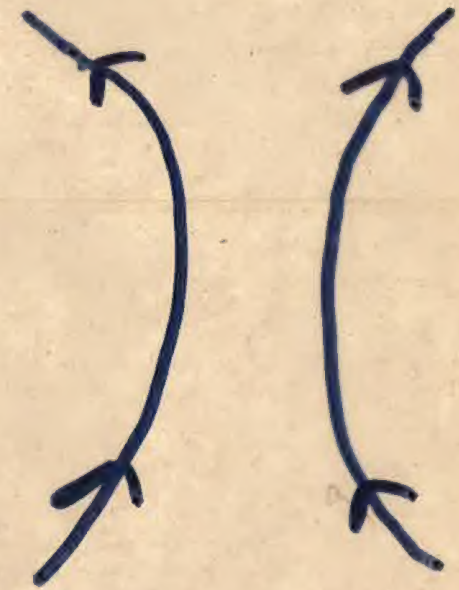
ANNIHILATION CREATION AND THE VACUUM



Does Exchange of Virtual
Particles always produce
Repulsion?



Attraction



Repulsion